Mathematical Biology -modeling by reaction-diffusion system-

Fall 2024

Instructor: Toshiyuki Ogawa

Textbooks:

- C.Cosner, "Reaction–Diffusion Equations and Ecological Modeling" in Tutorials in Mathematical Biosciences IV, Springer, 2008.
- Vladimir K Vanag, Irving R Epstein, "Pattern formation mechanisms in reaction-diffusion systems", Int J Dev Biol. 2009;53(5-6):673-81. doi: 10.1387/ijdb.072484vv.

Prerequisites: Basic knowledge of calculus, differential equations, and linear algebra. Familiarity with mathematical modeling concepts is advantageous but not mandatory. Mathematical techniques will be introduced as necessary.

Target Audience: This course is designed for students in mathematics, ecology, chemistry, and related fields interested in understanding the mathematical modeling and applications of reaction-diffusion equations.

Grading: Grades will be based on homework assignments and a final project. Homework will include both paper-and-pencil and programming assignments.

Course Description: Reaction-diffusion equations serve as powerful tools in understanding spatial patterns in diverse disciplines like ecology and chemical reactions. In this course, we delve into the mathematical framework of reaction-diffusion models, exploring their applications and analytical methods.

Course Objectives:

Provide an in-depth understanding of reaction-diffusion equations and their relevance in ecological and chemical systems.

Introduce the derivation of reaction-diffusion models and fundamental aspects of their analysis using partial differential equations and dynamical systems theory.

Explore mechanisms underlying spatiotemporal structure emergence and discuss recent advancements in the field.

Compensate for the limitations of mathematical analysis by introducing numerical computation methods and performing simulations.

Course Topics:

Introduction to Reaction-Diffusion Equations
- Interactions of species
- Diffusion and Random Walks
- Flux and boundary conditions

Solutions of Reaction–Diffusion Equations
- Linear Equations, Eigenvalue Problems
- Maximum Principles and Comparison Theorems
- Sub and Super solutions and Stability

Basic concept of dynamical system theory
- Invariant sets
- Stability of equilibrium
- Local and global Bifurcation
- Slow-fast system

Traveling wave
- Reduction to ODE by moving coordinate
- Propagation and Stability of Solutions
- Traveling wave for Lotka-Volterra competition diffusion system

Mechanisms of Spatiotemporal Structure Formation
- Turing instability
- Wave instability
- Instability from non-local effect

Chemical Reactions
- Belousov-Zhabotinsky reaction and its mathematical model

Numerical Analysis
- Numerical scheme for Reaction-diffusion equations
- Continuation from bifurcation point
- Continuation of heteroclinic or homoclinic orbits