

ECON 4150 (Fall 2025) Mathematical Economics

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Lectures: Two 1.5 hour lectures a week.

Office Hours: Monday at 10-11 am (some of these will be conducted via Zoom with notice). If you plan to attend *please let me know in advance* so as to manage congestion. If you are unable to make this time, feel free to arrange an appointment at a mutually convenient time.

Assessment: The grade will depend upon 6 homeworks, 3 in class open book exams.

Description

In a now famous passage of a 1906 letter from Alfred Marshall to Alfred Bowley, Marshall writes:

“I have not been able to lay my hands on any notes as to Mathematico-economics that would be of any use to you: and I have very indistinct memories of what I used to think on the subject. I never read mathematics now: in fact I have forgotten even how to integrate a good many things.

But I know I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules—(1) Use mathematics as a short-hand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can’t succeed in 4, burn 3. This last I did often.”

Bowley ignored the advice as did those who came after Marshall and rightly so. Economic problem often involve the interaction of many variables and mathematics is needed not just to express the relationships but to pin down their values. Further, purely verbal gymnastics often conceal implicit assumptions and jumps in logic. As a consequence, Mathematics has come to play a fundamental role in Economics.

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There are three basic mathematical questions that arise with unerring regularity in Economics:

Given a function f and a set S ,

- find an x such that $f(x)$ is in S . This is the **feasibility** question.
- Find an x in S that optimizes $f(x)$. This is the problem of **optimality**.
- Find an x in S such that $f(x) = x$; this is the **fixed point** problem.

The three problems are, in general, quite difficult. However, if one is prepared to make assumptions about the nature of the underlying function (say it is linear, convex or continuous) and the nature of the set S (convex, compact etc.) it is possible to provide answers and very nice ones at that.

This semester long course is about the answers, the relationships between them and how they arise in Economics. This will be illustrated with a host of examples. For example, how should options be priced, how should performance contracts be designed, how do we know Nash equilibria exist and how should one fairly divide the rent between roommates.

1. Week 1- 3: The theorem of the alternative and its applications.
2. Week 4-7: Convexity and the duality theorem of linear programming and its applications.
3. Week 8-10: Non-linear programs and their applications.
4. Week 11-15: Fixed point theorems and their application.

The grade for the course will depend upon seven homeworks an oral exam and a single take home final exam.

Much of what will be covered can be found in the Mathematical Appendix of **Microeconomic Theory** by Mas-Collel, Whinston and Green, Rangarajan Sundaram's **A First Course in Optimization Theory** or **Advanced Mathematical Economics** by your humble servant.

CANVAS is used to post announcements, homework assignments, video recordings and other important materials. You are responsible for regularly checking, downloading and reading materials posted on the site, as they form an integral part of the class.

Prerequisites: Ideally, Math 3120/3140, Math 3600, Econ 4100 and Econ 2100. However, one or more can be waived with permission. What is crucial is that

interested students have exposure to and comfort with the ‘theorem-proof’ style of mathematics.

Grade: It will depend on six homeworks (40 % of grade) and three in-class exams (60 % of grade). No scores are dropped.

Exams: No assistance may be given or received during an exam.¹ You are expected to abide by the Code of Academic Integrity in the completion of assignments, papers and exams.

Homework: Homework assignments to be submitted via CANVAS as a single PDF file on the due date. Use a scanning app like Dropbox or Genius to create the single PDF. **No late work is accepted.** There are *no* make-up homeworks. Students can be excused from at most two homeworks. The weights on the *subsequent* homeworks and final exam will be adjusted upwards to account for missing them.

Rules Regarding Assistance: Write-ups must be your original work. The use of materials containing solutions or partial solutions to the assignments (including solutions prepared by current or former students) would be contrary to Penn’s code of academic integrity.² If your solutions contains information from outside sources, you should properly acknowledge them.

While you are required to complete the assignments individually, I don’t wish to discourage learning from one’s peers. This leaves room for ambiguity, so I will try to make expectations as clear as possible. In brief:

1. Discussing the *general* ideas behind the problems is permitted.
2. Writing formal solutions should be *completely individual*, done in the equivalent of separate rooms.

As discussions of general ideas gradually become more specific, some judgment is unavoidable, but here’s the kind of interaction I have in mind: If a peer conveys an idea which seems central to the solution, *do not write it down....immediately*. Approach the problem again on your own as if afresh, influenced by however much

¹The Economics Department Course Policies, which include rules about exam attendance, make-up exams, grading appeals, etc., are available at: <http://economics.sas.upenn.edu/undergraduate-program/course-information/guidelines/policies>

²<https://catalog.upenn.edu/pennbook/code-of-academic-integrity/>

of their idea you remember. If you can re-create it without notes, you have mastered it, and I'm happy to give you credit. In this way we can let everyone help each other learn, while steering a wide berth around simple copying.

I also don't wish to discourage the use of computer assistance such as WolframAlpha or ChatGPT is permitted, with a "Disclose and Detail" policy of citing external assistance and showing work.

Due Dates TBD