

Syllabus: MATH 3130 Computational Linear Algebra

Instructor: John Green

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Class time: Tuesdays & Thursdays, , Room TBD, 10:15 a.m.

Office hours: TBD

NOTE: I am uploading this syllabus now so that you have an understanding of what to expect from the course, however, it is pending course preparations and other specifics that cannot be finalized until next semester. Additional information and specifics will be given then.

Course description: The subject of “Linear Algebra”, as its name may suggest, has its origins within the study of linear systems of algebraic equations. However, there is far, far more to it than that!

Over the course of the semester, we will develop the fundamental theories in linear algebra, and along the way explore connections with geometry in Euclidean space and beyond, paths in network graphs, bit-wise operations and symmetries in systems and some optimization problems, with many more applications out there to be explored.

The course is most recommended for students interested in subjects making use of math rather than those intending to focus primarily on math in their later studies. However, it is essential that the mathematical theories be explored accurately and with sufficient depth, so this will be the main focus of our course, though with many motivating applications and examples throughout.

Our primary perspectives will be with a view to the intuition behind key concepts and to computational techniques. Thus, we will not always prove major theorems, but will always endeavor to motivate them. Proofs will generally be included where they are algorithmic in nature, and thus can be leveraged to construct methods of computation.

Please note also that the descriptor “computational” means “relating to computation” – not coding for a computer! While we will certainly adopt a perspective highly applicable to computer science, the goal of the course is not to teach you how to implement the methods in code – though the skills we develop will certainly be highly adaptable.

Pre-requisites: Calculus up to MATH 1410 will be assumed, but will feature infrequently. The bulk of the course need only assume high-school level algebra, but ability to reason mathematically at a university level is essential. A sufficiently dedicated student who has taken any MATH course at Penn should be able to adjust.

Readings: I'll be providing a PDF of lecture notes written for the course as we proceed. No supplemental materials are necessary, but a wide range of books are out there should you need them – though I personally do not have any recommendations.

Grading: Anticipated to consist of weekly or biweekly homeworks together with a final exam, but the exact nature of this will depend on course enrollment and the number of dedicated graders.

Possible list of topics:

Core topics:

- Vector spaces and subspaces, linear transformations. Correspondence with matrix algebra in finite dimensions. Connection with linear systems of equations.
- Linear independence, span, bases and coordinates. Sifting algorithm.
- Image and kernel, rank-nullity theorem. Matrix row rank and column rank. Dual spaces.
- Gaussian elimination/row reduction. Inverse matrices via elementary row and column operations/elementary matrices.
- Determinants as a signed volume via elementary matrices. Definition of determinant using permutation groups. Cofactor expansion method, adjugate matrix, Cramer's Rule.
- Change of basis/coordinates. Equivalent and similar matrices. Eigenvalues/eigenvectors for linear operators.
- Matrix diagonalization. Jordan canonical form and statement of existence theorem with calculation methods.
- Bilinear maps/forms. Change of basis via transpose, symmetric maps, inner products.
- Orthogonal matrices. Norms from inner products, preservation of distance and angle. Diagonalization of symmetric matrices. Positive definite matrices.
- Unitary, Hermitian and normal matrices.
- Basics of multilinear algebra, tensor products, wedge products, alternating forms. Connection with cross product.

Bonus topics/applications, subject to pruning:

- Quadratic forms and classification of quadric surfaces.
- Matrix groups. General linear, special linear, orthogonal, special orthogonal, unitary, special unitary.
- Matrix exponential. Application to linear systems of ODEs. Linear difference equations.
- Polar, Gram/QR, Singular Value and LU decompositions.
- Least squares optimization. Positive-definite Hessians and convex functions, convex optimization introduced.
- Paths in graphs, Markov chains.

- Fourier transform as eigenvector decomposition. Discrete Fourier transform.
- General norms/metrics. Frobenius norm.
- Infinite-dimensional function spaces. Definition of completeness.
- Contraction mapping principle as a basis for numerical algorithms. Error bounds in Jacobi's method, Newton-Raphson Iteration. Computational expense.