Syllabus: MATH 3600 Advanced Calculus

Instructor: John Green

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Class time: Mondays & Wednesdays, Annenberg School ANNS 111, 12:00 p.m.

Recitations: DRLB 2C8, Tuesday (section 101) or Thursday (section 102), DRLB 2C8, 7:00 p.m.

Office hours: Will be scheduled according to what availability works best for the class and posted on the Canvas page.

Course description: Calculus is an invaluable tool for understanding continuous phenomena. However, this sophisticated theory is not so easy to rigorously formalize, and without such a framework it becomes very easy for contradictory conclusions and poorly reasoned claims to plague our mathematics. The field of Mathematical Analysis is the solution to these problems, and the goal of this course is to develop its basic principles from the ground up.

We will start by reviewing carefully the concepts of sets and numbers, and consider the real numbers axiomatically. Then, we will discuss with infinite sequences, asking questions like "What does it mean for something to be the limit of a convergent sequence?". This topic is not only a great place to ease into the concepts of analysis, but is also necessary to develop tools needed for all our later topics, such as infinite series and integration. We will then discuss continuous functions of a single real variable, differentiation (the infinitesimal rate of change of a function) and integration (a tool with far too many interpretations to describe here!). Along the way we will study many other topics such as Taylor series and uniform convergence.

This is a challenging but deeply rewarding course. Don't expect to understand everything perfectly the first time you see it – this is stuff that everyone takes time to process at first, and most people find that it takes a few weeks to feel like they've gotten into the right mindset for Analysis. It will require perseverance, but is well worth the effort.

Pre-requisites: As we will be building our theory from the ground up, essentially no background knowledge is required, though the concepts we address will be challenging and a strong logical skill set is crucial. MATH 1400 Calculus I or equivalent is not strictly necessary, but provides very valuable context and is highly recommended.

Who is this course for? Analysis is one of the fundamental areas of Mathematics and should be studied by anyone considering studying further math. Additionally, its precise formulations are key to more advanced topics with applications outside of math, such as numerical analysis in Engineering, and metric spaces in Economics.

Beyond this, the mindset required for Analysis, of being able to frame "intuitive" phenomena within a logical framework where precise reasoning can be applied, provides excellent training for all kinds of problem solving. Students in Philosophy may find this particularly meaningful.

Readings: I'll be providing a PDF of typed lecture notes written for the course as we proceed, and no supplemental readings are necessary. If you would like more materials, see the section "Help with the course" below.

Grading: Homework assignments (approx. weekly) – 40%, Final exam – 60%.

Homework policies: Assignments will be given approximately once per week. No late submissions will be accepted, instead, we will drop the lowest scores from the first and second halves of the semester to mitigate missed assignments due to illness or other circumstances. Full policies will be described on the course Canvas page.

Help with the course: If you are struggling to understand any of the material or any homework problems, I recommend trying this (in this order):

1. Try rereading/thinking about it for a while. The more conclusions/realizations that can be reached as a result of your own thoughts, the better.
2. Ask your peers. Working on your understanding/working through problems via mutual collaboration with others in the same position is an excellent way to learn mathematics.
3. Ask your TA during recitation, or ask me during office hours.

In particular, whilst internet resources and textbooks can be an excellent supplement, I encourage *not* relying on them in general – besides the abundance of erroneous information, reading several different explanations of the same concept makes it easy to *feel* like you've learned a lot more than you have!

As a way to limit that, if you feel that the resources provided are insufficient even after struggling with them for the first few weeks, I recommend choosing just one additional book. Some possible options are Spivak's "Calculus", Apostol's "Calculus Vol. 1: One-Variable Calculus, with an Introduction to Linear Algebra" and Rudin's "Principles of Mathematical Analysis", but I do not have any particular recommendation for any of these.

Further help: If you are experiencing significant problems, be it consistently poor performance even after several weeks, time taken off due to personal issues, or anything else, I'll do what I can to help. Your physical and mental well-being are important, so if you find yourself needing to take time off, experiencing exam anxiety, etc, I will do my best to find a solution that works for you. In the first instance, please send me an email and I'll see what I can do.