

# Syllabus: MATH 3610 Advanced Calculus II

**Instructor:** John Green

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**Class time:** Mondays & Wednesdays, CHEM B13, 12:00-1:30 p.m.

**Recitations:** Tuesday, DRLB 3C8 (section 103) or Thursday, DRLB 2C2 (section 104), 7:00-8:00 p.m.

**Office hours:** Mondays & Tuesdays, 4:00-5:00 p.m., subject to change or additional offerings as necessary.

## Pre-requisites:

- **Essential:** MATH 3600. This course builds directly on top of the material covered in MATH 3600. It will be assumed that you are comfortable with understanding the subtleties of rigorous proof in mathematical analysis, and have some experience constructing your own arguments. We will further build and cement these skills here.

Topics covered in MATH 3600 change depending on the year and instructor, and we have a non-trivial number of people who took MATH 3600 in a session other than Fall 2024, so I've designed this course in such a way that only some core topics need be assumed. In particular, the following are strictly assumed:

1. Basics of mathematical reasoning and set notations,
2. Properties of the real numbers, including completeness,
3. Convergence of sequences and series of real numbers,
4. Continuity, limits and derivatives for real-valued functions of a single real variable,
5. Intermediate, extreme and mean value theorems,
6. Definitions of exponentials, logarithms, real number exponents and trigonometric functions, and their basic properties.

In the interest of providing motivation for several concepts, I may often talk as if you have studied the following additional topics, although I have structured the course so as to be logically independent of these topics:

1. Power series and their properties,
2. Taylor's Theorem,

3. Uniform convergence,
4. Integration of continuous functions on closed intervals,
5. Fundamental Theorem of Calculus.

I will provide lecture notes from when I taught MATH 3600 in Fall 2023 for reference on all of the above, although this should not be strictly necessary.

If you are not sure whether you know enough about a particular topic, or unsure if it is acceptable to assume a particular result or property, please do ask!

- **Recommended:** MATH 1410, MATH 3140, MATH 2400. We will formally require a minimum of linear algebra experience – you should know what a vector space over the real or complex numbers is, what a linear map is, and how to multiply matrices. Determinants, and their relation to volumes in  $\mathbb{R}^d$ , will also play a role. For this, MATH 3140 is ideal, but 2400, 3120 or 3130 are both more than sufficient.

No experience with multivariable calculus is strictly required, but some prior familiarity with multivariate differentiation and integration will provide valuable motivation.

**Readings:** None required! I'll be providing a PDF of typed lecture notes written for the course as we proceed, and no supplemental readings are necessary.

**Course description:** If you're planning to take this course, you should already have finished a first semester in analysis. The kinds of theorems we are interested in – those concerning convergence, continuity, limits, derivatives and integrals – are much the same here, as are the principles of mathematical logic we will use to prove them. What is new is the scope and nature of our theory!

We will begin by extending ideas of convergence and continuity not just to  $\mathbb{R}^d$ , but to the abstract setting of metric spaces. This generalization will quickly pay off, as we use this concept to define distances not just between points in Euclidean space, but between functions – and then we can leverage our intuitions about Euclidean distances to provide proofs of results such as the Implicit Function Theorem.

We will also introduce some of the ideas of metric space topology which will be important in many of our arguments – that of open and closed sets, of compactness, completeness and more. All this will be necessary for exploring key results on higher-dimensional derivatives and integrals, culminating in a rigorous theoretical framework for multivariable calculus and beyond.

As a student, the core goals of this course will be for you to develop your proof skills and to become familiar with the modern theories in analysis which are essential to further learning.

**Time permitting...** It is expected that the above topics will fill the semester, but we may have time to briefly explore some extra (non-examinable!) topics, subject to interest. Possible topics include: Further results in metric spaces, such as the Arzelà-Ascoli Theorem, Topics in Functional analysis (Hilbert spaces, Examples of Banach Algebras), Existence & Uniqueness for Ordinary Differential Equations, Fourier analysis, Manifolds, Optimization, Fractals.

**Course structure:** In a typical week, you will attend two lectures and one recitation. The lectures will be fairly traditional – we will introduce and discuss the course topics at a contemplative pace,

with plenty of opportunity to ask questions and clarifications. You should then expect to review the material further, and bring any lingering questions to office hours and recitation.

The recitations will be mainly for working on homework problems in a collaborative setting, with some worked examples provided. My apologies for the lateness of the recitation – there’s nothing I can do about this – but attending both lectures and recitations regularly will be essential to your success!

**Grading:** Homework assignments (weekly) – 40%, Two Midterms – 20%, Final exam – 35%, Progress meeting – 5%.

**Homework policies:** There will be 12 homeworks set each week on Monday, starting January 27<sup>th</sup>, due the next Monday, with the last due on April 28<sup>th</sup>, and with a one week offset occurring during the Spring break. No late submissions will be accepted, but to account for this, the lowest two scores will be dropped to account for missed assignments due to illness or other circumstances. See also “Extenuating circumstances”.

You are welcome to collaborate with other students, and encouraged to do so both in recitations and outside of them, but your written solutions should be your own. Please also note the names of anyone you collaborate with on your submission.

**Midterms:** There will be two in-class midterms, each lasting one hour, tentatively scheduled for February 19<sup>th</sup> and March 26<sup>th</sup>. No note sheets or calculators. Please notify me as soon as possible of any potential conflicts/issues.

**Final:** The final exam will be cumulative, with a slight weighting towards material covered after the second midterm. Again, no note sheets.

**Progress meeting:** I will arrange for everyone to have a progress meeting soon after the first midterm, so that I can get to know each of you a bit better, how the course is going for you and to make sure your studies are on the right track!

**Accommodations & Accessibility:** Things such as extra time for exams should be arranged directly with SDS as usual, and I will do whatever is necessary to accommodate. However, please don’t hesitate to ask for any assistance beyond this, such as providing course materials like lecture notes and homeworks in accessible formats.

Let me know what you need: Be it grey page backgrounds to reduce the eye strain of screen viewing or text-to-speech readability, I will do my best to find a solution.

**Extenuating circumstances:** If you experience any issues which may substantially affect your ability to engage with the course, such as extended illness, death in the family or other personal issue, please reach out as soon as you are able so that we can work together to figure out what help you need. These course policies are calibrated to normal circumstances, and I will gladly modify them on a case-by-case basis subject to personal need.