Math 5861: Mathematical Modeling in Biology
Instructor: Daniel Cooney
Fall 2022

1 Course Description

This course will provide an introduction to mathematical modeling of biological populations, with an emphasis on applications to ecology, evolutionary biology, and infectious disease dynamics. Mathematical techniques from dynamical systems, stochastic processes, and partial differential equations will be introduced to model a range of finite and infinite populations with well-mixed or spatially-explicit population structures. The first half of the course will focus on evolutionary game theory, exploring the evolution of cooperative behaviors and applications from mathematical oncology to modeling social systems. The second half of the course will emphasize spatial models in ecology, highlighting traveling wave and pattern formation emerging from collective behaviors of interacting populations.

2 Prerequisites

The main prerequisite for the course is familiarity with linear algebra and ordinary differential equations at the level of Math 240. The course will also use partial differential equations (Math 241, Math 425) and probability (Math 430), but relevant concepts from these topics will be introduced as needed.

3 Assignments and Grading

The course will have the following graded work.

- 7-8 problem sets assigned on an approximately biweekly basis
- An in-class midterm exam (scheduled for the week of October 10)
- A final project/paper, which can consist of an expository paper, literature review, or a preliminary research report (can be completed individually or in a group of 2-3 students)

The breakdown of grades will be 40% homework, 20% midterm exam, 30% final project, and 10% participation.

4 Textbook

There will be no required textbook for the course. The following books are useful references for portions of the course material.

- Edelstein-Keshet, L. (2005). *Mathematical Models in Biology.* SIAM.
- Hofbauer, J. & Sigmund, K. (1998). *Evolutionary Games and Population Dynamics.* Cambridge University Press.
- Kot, M. (2001). *Elements of Mathematical Ecology.* Cambridge University Press.
- Murray, J. D. (2002). *Mathematical Biology I. An Introduction.* Springer.
- Nowak, M. A. (2006). *Evolutionary Dynamics: Exploring the Equations of Life.* Harvard University Press.
- Sandholm, W. H. (2010). *Population Games and Evolutionary Dynamics.* MIT Press.

Optional reading assignments will accompany each lecture (see course outline), which may be useful for picking a final project topic.

5 Course Outline

- Week 1 (8/29 - 9/4)
  - Lecture 1: The Evolution of Cooperation
  - Lecture 2: Evolutionary Game Theory and Derivation of Replicator Equations
  - Reading
    * Axelrod, R. & Hamilton, W. D. (1981). The evolution of cooperation. *Science*, 211(4489), 1390-1396.
    * Hofbauer, J. & Sigmund, K. (2003). Evolutionary game dynamics. *Bulletin of the American Mathematical Society*, 40(4), 479-519.
    * Lahkar, R. & Sandholm, W. H. (2008). The projection dynamic and the geometry of population games. *Games and Economic Behavior*, 64(2), 565-590.
- Week 2 (9/5 - 9/11)
  - Lecture 3: Evolutionary Stable Strategies and Long-Time Behavior of Replicator Dynamics
  - Lecture 4: ODE Models for Ecological Dynamics (Lotka-Volterra, Allee Effects)
  - Reading
    * Taylor, P. D. & Jonker, L. B. (1978). Evolutionary stable strategies and game dynamics. *Mathematical Biosciences*, 40(1-2), 145-156.
    * Pascual, M., Roy, M., & Laneri, K. (2011). Simple models for complex systems: exploiting the relationship between local and global densities. *Theoretical Ecology*, 4(2), 211-222.
    * May, R. M. & Leonard, W. J. (1975). Nonlinear aspects of competition between three species. *SIAM Journal on Applied Mathematics*, 29(2), 243-253.
- Week 3 (9/12 - 9/18)
  - Lecture 5: Modeling Infectious Disease Dynamics
  - Lecture 6: Global Stability of Infectious Disease Models
  - Reading
    * Weiss, H. H. (2013). The SIR model and the foundations of public health. *Materials Matemátics*, 0001-17.
    * Feng, Z. (2007). Final and peak epidemic sizes for SEIR models with quarantine and isolation. *Mathematical Biosciences & Engineering*, 4(4), 675.
    * Hethcote, H. W. (1976). Qualitative analyses of communicable disease models. *Mathematical Biosciences*, 28(3-4), 335-356.
- Week 4 (9/19 - 9/25)
  - Lecture 7: Evolutionary Dynamics in Finite Populations
  - Lecture 8: Evolutionary of Cooperation in Finite Populations
  - Reading
    * Taylor, C., Fudenberg, D., Sasaki, A., & Nowak, M. A. (2004). Evolutionary game dynamics in finite populations. *Bulletin of Mathematical Biology*, 66(6), 1621-1644.
    * Traulsen, A. & Hauert, C. (2009). Stochastic evolutionary game dynamics. *Reviews of Nonlinear Dynamics and Complexity*, 2, 25-61.
- Week 5 (9/26 - 10/2)
  - Lecture 9: Evolutionary Graph Theory
  - Lecture 10: Games on Graphs

- Reading
  - Lieberman, E., Hauert, C., & Nowak, M. A. (2005). Evolutionary dynamics on graphs. *Nature*, 433(7023), 312-316.
  - Ohtsuki, H., Hauert, C., Lieberman, E., & Nowak, M. A. (2006). A simple rule for the evolution of cooperation on graphs and social networks. *Nature*, 441(7092), 502-505.

- Week 6 (10/3 - 10/9)
  - Lecture 11: Cooperation in Repeated Games: Direct Reciprocity
  - Reading
    - Posch, M. (1999). Win–stay, lose–shift strategies for repeated games—memory length, aspiration levels and noise. *Journal of Theoretical Biology*, 198(2), 183-195.
    - Stewart, A. J. & Plotkin, J. B. (2013). From extortion to generosity, evolution in the iterated prisoner's dilemma. *Proceedings of the National Academy of Sciences*, 110(38), 15348-15353.

- Week 7 (10/10 - 10/16)
  - Lecture 12: Evolution in Structured Populations
  - Midterm Exam
  - Reading
    - Tarnita, C. E., Ohtsuki, H., Antal, T., Fu, F., & Nowak, M. A. (2009). Strategy selection in structured populations. *Journal of Theoretical Biology*, 259(3), 570-581.
    - Nowak, M. A., Tarnita, C. E., & Antal, T. (2010). Evolutionary dynamics in structured populations. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 365(1537), 19-30.

- Week 8 (10/17 - 10/23)
  - Lecture 13: Evolutionary Games and Social Contracts
  - Lecture 14: Evolutionary Dynamics of Cancer
  - Reading
    - Binmore, K. (1990). Evolution and utilitarianism: Social contract III. *Constitutional Political Economy*, 1(2), 1-26.
    - Aktipis, C. A., Boddy, A. M., Jansen, G., Hibner, U., Hochberg, M. E., Maley, C. C., & Wilkinson, G. S. (2015). Cancer across the tree of life: cooperation and cheating in multicellularity. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 370(1673), 20140219.
    - Pacheco, J. M., Santos, F. C., & Dingli, D. (2014). The ecology of cancer from an evolutionary game theory perspective. *Interface Focus*, 4(4), 20140019.

- Week 9 (10/24 - 10/30)
  - Lecture 15: Continuous Strategy Games: Adaptive Dynamics
  - Lecture 16: From Stochastic Spatial Models to Reaction-Diffusion-Advection Equations
  - Reading
    - Brännström, Å., Johansson, J., & Von Festenberg, N. (2013). The hitchhiker's guide to adaptive dynamics. *Games*, 4(3), 304-328.
    - Brush, E. R., Leonard, N. E., & Levin, S. A. (2016). The content and availability of information affects the evolution of social-information gathering strategies. *Theoretical Ecology*, 9(4), 455-476.
    - Durrett, R., & Levin, S. (1994). The importance of being discrete (and spatial). *Theoretical Population Biology*, 46(3), 363-394.

- Week 10 (10/31 - 11/6)

- Lecture 17: PDE Solution Techniques: Separation of Variables and Method of Characteristics
- Lecture 18: PDE Models in Ecology: Critical Patch Sizes
- Reading
  - Holmes, E. E., Lewis, M. A., Banks, J. E., & Veit, R. R. (1994). Partial differential equations in ecology: spatial interactions and population dynamics. *Ecology*, 75(1), 17-29.
  - Briggs, C. J. & Hoopes, M. F. (2004). Stabilizing effects in spatial parasitoid–host and predator–prey models: a review. *Theoretical Population Biology*, 65(3), 299-315.

- Week 11 (11/7 - 11/13)
  - Lecture 19: Traveling Waves in Ecology and Evolution
  - Lecture 20: Diffusion and Telegraph Models of Animal Movement
  - Reading
    - Holmes, E. E. (1993). Are diffusion models too simple? A comparison with telegraph models of invasion. The American Naturalist, 142(5), 779-795.
    - Mollison, D. (1991). Dependence of epidemic and population velocities on basic parameters. *Mathematical Biosciences*, 107(2), 255-287.
    - Bernoff, A. J., Culshaw-Maurer, M., Everett, R. A., Hohn, M. E., Strickland, W. C., & Weinburd, J. (2020). Agent-based and continuous models of hopper bands for the Australian plague locust: How resource consumption mediates pulse formation and geometry. *PLoS Computational Biology*, 16(5), e1007820.

- Week 12 (11/14 - 11/20)
  - Lecture 21: Turing Instability in Predator-Prey Models
  - Lecture 22: Chemotaxis and Collective Motion
  - Reading
    - Segel, L. A. & Jackson, J. L. (1972). Dissipative structure: an explanation and an ecological example. *Journal of Theoretical Biology*, 37(3), 545-559.
    - Murray, J. D. (1990). Discussion: Turing's theory of morphogenesis—its influence on modelling biological pattern and form. *Bulletin of Mathematical Biology*, 52(1-2), 117-152.
    - Keller, E. F. & Segel, L. A. (1970). Initiation of slime mold aggregation viewed as an instability. *Journal of Theoretical Biology*, 26(3), 399-415.
    - Topaz, C. M., Bertozzi, A. L., & Lewis, M. A. (2006). A nonlocal continuum model for biological aggregation. *Bulletin of Mathematical Biology*, 68(7), 1601.

- Week 13 (11/21 - 11/27)
  - Lecture 24: The Kimura Equation and Population Genetics
  - Reading
    - Etheridge, A. Diffusion Process Models in Mathematical Genetics.
    - Traulsen, A., Claussen, J. C., & Hauert, C. (2005). Coevolutionary dynamics: from finite to infinite populations. *Physical Review Letters*, 95(23), 238701.

- Week 14 (11/28 - 12/4)
  - Lecture 25: PDE Models of Multilevel Selection
  - Lecture 26: Landscape Dynamics: PDE Models of Continuous Strategy Games
  - Reading
    - Luo, S. (2014). A unifying framework reveals key properties of multilevel selection. *Journal of Theoretical Biology*, 341, 41-52.

        * Luo, S. & Mattingly, J. C. (2017). Scaling limits of a model for selection at two scales. *Nonlinearity*, 30(4), 1682.
        * Johnson, J. D., White, N. L., Kangabire, A., & Abrams, D. M. (2021). A dynamical model for the origin of anisogamy. *Journal of Theoretical Biology*, 521, 110669.

- Week 15 (12/5 - 12/11)
    - Lecture 27: Final Presentations 1
    - Lecture 28: Final Presentations 2
- Week 16 (12/12)
    - Lecture 29: Multilevel Selection in Evolutionary Games