

Math 5080: Advanced Analysis

Instructor: Philip T. Gressman

Course Description

508. ADVANCED ANALYSIS. Terms: A

Staff. Prerequisite(s): Math 2400/2410 or 2600. A previous course with proofs (e.g. Math 1610, 2600, 2020, 2030) is recommended. Credit given for each semester.

Construction of the real numbers, the topology of the real line and the foundations of single variable calculus. Notions of convergence for sequences of functions. Basic approximation theorems for continuous functions and rigorous treatment of elementary transcendental functions. The course is intended to teach students how to read and construct rigorous formal proofs. A more theoretical course than Math 3600.

The main differences between this course and Math 3600:

- **More Material:** There are many topics in analysis which are useful to know for graduate school but tend not to be included in typical undergraduate analysis courses. This course includes as many of these “hidden curriculum” topics as possible in addition to the “big theorems” of elementary real analysis.
- **Higher Expectations for Engagement:** It is assumed that students in this course will be learning new ideas in many different contexts: reading, watching video, extrapolating ideas through assigned work, participating in individual and group in-class exercises. Many ideas will be encountered in only one of the various contexts just mentioned (e.g., some new ideas will be explored in the homework without first being discussed in a lecture).
- **Higher Grading Standards:** The purpose of this course is to prepare students for graduate school in mathematics. In this context, excellent work should be not only free of even minor errors, but also clear and concise (i.e., not just correct, but also well-written). In comparison, most undergraduate courses are much more tolerant of unclear writing (for example).

Course Format

Major elements of the course include:

- **Preparatory Videos:** Approximately 45–60 minutes of prerecorded, fully asynchronous video content per week. Videos should be watched before the relevant class meeting (following the structure of each Canvas module).
- **In-Person Classes:** Lecture time will generally be split into two parts:
 - **Instruction Sessions (TR 10:15am–11:00am)** for lectures, in-class group work/active learning exercises
 - **Question/Work Sessions (TR 11:00am–11:45am)** for discussion of previous or upcoming homework assignments
- **Weekly Lab:** Mondays or Wednesdays 7:00pm–8:15pm depending on lab registration. Details of lab organization, schedule, and grading will be set by our teaching assistant.
- **Homework:** Most weeks there will be a traditional homework assignment, typically due by 7:00pm on Friday. Late work will not be accepted unless there are extenuating circumstances like severe illness. Extension requests should be sent directly to our teaching assistant. **All submitted work in this class must be typed in \LaTeX** ; handwritten work (including scans and pictures of handwritten work) will not be accepted for grading. Homework must be submitted by uploading your PDF file to Canvas. The pdf and the file name should include your name and the homework number.
- **Biweekly Assessment Items:** Roughly biweekly, there will be either a timed Canvas quiz or a “Prelim Problem.” The Canvas quiz will be open for a 3-day window (usually Tuesday through Thursday). The quiz will have a 65-minute time limit; students must type a response in \LaTeX , and upload the PDF output before time expires. For quizzes, students are not allowed to use any reference materials or receive any help from any source. Prelim Problems are untimed; students may use reference materials but may not seek help from classmates or peers.
- **Reflections:** Approximately every other week, students will be asked to submit a brief canvas survey quiz to answer questions and reflect upon recent work and experiences in the course.
- **Oral Final Exam:** Each student will meet with the instructor for 30 minutes at the end of the semester to discuss a series of mostly conceptual questions about the content of the entire course (i.e., the exam is cumulative). More details about the oral exam process will be provided near the end of the semester.

There will not be any traditional written exams in this course.

Textbooks

No textbook is formally required, but our schedule will be organized to align with the structure of the book Understanding Analysis by Stephen Abbott, which you can download for free from the publisher's website here: <https://link.springer.com/book/10.1007/978-1-4939-2712-8>. Students should also consult Lebl's online textbook Basic Analysis: Introduction to Real Analysis <https://www.jirka.org/ra/>. A third important reference for later material is Fourier Analysis: An Introduction by Stein and Shakarchi.

Important Websites

Canvas: Course materials will be posted on Canvas: <https://upenn.instructure.com/>.

Notes and Videos: The instructor maintains a website with preparatory videos and notes covering all aspects of this course. Links are available from within Canvas modules, but it may be useful to independently bookmark the site: <https://www2.math.upenn.edu/~gressman/analysis/>

Grade Components

The relative weights which, in nearly all cases, will be used to combine individual grades into the final course grade are as follows.

- **(5%) Preparatory Videos and In-Person Classes** including items such as short "prep quizzes," group work assignments, etc. Items in this category are graded for completion and participation/demonstrated good-faith effort. Not graded for correctness.
- **(10%) Weekly Lab** details TBA from the teaching assistant.
- **(10%) Homework**
- **(40%) Quizzes and Prelim Problems** divided roughly equally among all assignments in this category.
- **(5%) Reflections** graded for participation/completion and demonstrated effort.
- **(30%) Final Oral Exam**

Grades are independently computed via multiple algorithms to ensure robustness of the final grade to reasonably small changes in individual assignment scores.

Electronic Lecture Content

Beginning Fall 2022, in-person lectures will no longer be recorded or live-streamed. Exceptions will be made as necessary (if, for example, the instructor becomes ill). Recordings of lectures from previous years are still available on Canvas. Current-semester digital lecture notes *will* be made available on Canvas shortly after each lecture.

Lecture Topics

See Canvas or the notes website (links above) for a complete listing of topics by day for this course.

Academic Integrity

It is assumed that students in this graduate-level course have an advanced awareness of the Code of Academic Integrity: <https://catalog.upenn.edu/pennbook/code-of-academic-integrity/>. Any violation of the Code, whether it be on homework assignments, quizzes, during the final oral exam, or at any other point in the semester, will be taken extremely seriously. Remedies to restore fairness of the grading standards in this class may include revoking some or all credit for the affected assignment. Students may also be referred to the Office of Student Conduct for further disciplinary proceedings.

COVID-19/Illness Policies

Although most students are at low risk for serious illness or death, all students are strongly encouraged to remember the many different ways that COVID continues to impose significant burdens of many different kinds in very unequal ways. Each of us makes choices that impact not only ourselves, but also those around us and, by extension, all members of the University community.

- As of the start of the Fall 2022 semester, **students are expected to wear masks when attending in-person activities for this class**. Here "expected" includes the possibility that students without a mask may be asked to don one. During lecture time, disposable masks will be available for those who occasionally forget to bring their own.
- **Any student who suspects that they may be ill is expected to refrain from attending in-person activities for this class** until they are confident that they do not have a major illness (including but not limited to COVID-19 and the flu) which could significantly impact others.
- Students who are incapacitated by illness of any kind should inform the instructor as soon as possible. **Students are expected to manage their work in this course responsibly to mitigate the foreseeable impact of potential illness**. What this means in practice is that students who fall ill immediately before a deadline (e.g., the day that a homework assignment is due) should expect the accommodations provided to be proportional to the demonstrated responsibility in planning (e.g., less flexibility will be extended to the student who hasn't yet started the homework).