Math 5861 / Biol 5860: Mathematical Modeling in Biology: Cell Motility

Instructor: Albane Thery

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This course focuses on cellular motility in fluids, a domain at the intersection of physical and natural sciences. It will use the mathematical framework of viscous fluids and random walks to model the physical constraints and strategies relevant to microscale locomotion.

Course Description

We introduce the mathematical framework that describes the motion of swimming cells, in particular microorganisms such as bacteria, algae or spermatozoa, in viscous fluids. Microswimmers are confronted to two important physical challenges: they move in a viscous fluid, which strongly constrains which movements lead to net propulsion, and they are exposed to noise and fluctuations. First, we will study inertia-less swimming, and the range of relevant biophysical phenomena from the individual to the population-wide scale. The second part of the course will introduce strategies to navigate noisy environments, such as run-and-tumble dynamics, and to search for resources, such as chemotaxis.

Prerequisites

This course is open to undergraduate and graduate students from any department, with an interest in biophysics or fluid mechanics. Being by essence interdisciplinary, this course is intended to introduce students in mathematical fields to the modelling of living systems, and to introduce students in the life sciences to a mathematical approach to understanding living systems. Some familiarity with differential equations and basic fluid mechanics is helpful for background knowledge, in particular the derivation of the mass and momentum conservation equations, but an introduction to the modelling of viscous flows and diffusion will be provided. Undergraduate knowledge of mathematical methods, as well as vector calculus, will be assumed.

Course Highlights

- Introduction to an interdisciplinary, theoretical approach to studying biological and fluid dynamics problems.
- Lecture material and optional readings will draw on both classical results in cell motility (flagellar propulsion, run-and-tumble dynamics, chemotaxis) and on modern research topics studying microorganisms (from bacterial contamination to collective dynamics of large populations of swimmers)
- We will derive the mathematical framework to model the locomotion of microswimmers from first principles, thus showing how the tools of applied mathematics and continuum mechanics can provide quantitative and physical insights into the biological world.
- We will highlight the many experimental applications of the models we use. This course will show how research can progress from a dialogue between experimental data stemming from progresses in microscopy and microfluidics and physical models.
- Final project will allow students (either individually or in groups) to delve deeper into research questions either through a literature review or develop their own approach for a biology-inspired physical problem through a preliminary modelling project.

Assignments and Grading

The course will have the following graded work.

- 7-8 problem sets assigned on an approximately biweekly basis
- A take-home midterm exam (scheduled for the week of October 10)
- A final project/paper, which can consist of an expository paper, literature review, or a preliminary research report (can be completed individually or in a group of 2-3 students)

The breakdown of grades will be 40% homework, 15% midterm exam, 35% final project, and 10% participation.

Textbook

There will be no required textbook for the course. The following books are useful references for portions of the course material.

- Lauga, E. (2022). *The Fluid Dynamics of Cell Motility* . Cambridge University Press.
- Lauga, E. (2022). *Fluid Mechanics (Very Short Introductions).* Oxford University Press.
- Berg, H. (2005). *E. coli in Motion* . Springer.
- Berg, H. (1993). *Random Walks in Biology* . Princeton University Press.

Optional reading assignments will accompany each lecture, which may be useful for picking a final project topic.