Physics 5516: Electromagnetic Phenomena

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> 27 January 1884. Thought about electromagnetic rays.
> 11 May. Hard at work on Maxwellian electromagnetics.
> 13 May. Nothing but electromagnetics.
> 8 July. Electromagnetics, still without success.
> 17 July. Depressed; could not get on with anything.
> 24 July. Did not feel like working.
> 7 August. Saw from Ries's book that most of what I have found so far is already known.
> – From the Diary of Heinrich Hertz

Information below is subject to change as the term progresses.

Our goal in this course is to examine some electromagnetic phenomena relevant to the research being done in this department, and obtain them as consequences of classical electrodynamics. We want to look at the phenomena through the lens of mathematics, to see common themes. Finally, we want to consolidate many bits of information you've gathered in your previous courses into a more coherent whole.

Most of the students will be Physics PhD students. Undergraduates are welcome, but understand that this will be a fast ride assuming that you have taken PHYS360–361 or are willing to catch up.

Announcements and communication: `canvas.upenn.edu/`. Once you have registered, please log into Canvas and check that you have access. Instead of the Canvas message system, I will send you e-mail to the address that the registrar has on file for you, or to any other address you like (but you'll have to tell me).

Written work:
May be submitted in hard copy or electronically via Canvas.

Assessment:
There will be one in-class midterm exam. Later there will be one midterm project. There will also be a final project. There will be no in-class final exam. The two projects will have a window of several days from getting the problem to submission, and *firm deadlines.* My intention is that, if you have been staying current with all the reading and other assignments, then each project will require only a few hours, giving you flexibility.
There will also be a few shorter, "ordinary" problem sets roughly biweekly.

Office hours: We'll arrange these after classes begin to suit the class.

General policies: see separate "Course Policies" handout.

General prerequisites:
Familiarity with Maxwell equations in differential and integral form. Familiarity with electro- and magnetostatics. Familiarity with special relativity and some linear algebra, at

the levels of PHYS3361–3362 and MATH2400. Facility with some computer-math package, such as Python, but you can use whatever package you are comfortable with.

Books (more are listed in the References):
The main text will be supplied electronically, for free, via Canvas. SEAS copy center will sell you a hard copy at a bargain price.
You may find useful *A Student's Guide to Python for Physical Modeling* Second Edition, by JM Kinder and P Nelson (Princeton U Press 2021). Hard copy is available at the Penn bookstore; the e-book is available on many platforms (listed at `press.princeton.edu/books`).
Other required reading, if any, will be supplied via Canvas.
Apart from the main text, my favorite all-round backup is Pollack and Stump, *Electromagnetism* (Addison Wesley, 2002), available on reserve at the library.
Also useful (but not required):
D. Fleisch, *A student's guide to vectors and tensors*
D. Fleisch, *A student's guide to Maxwell's equations*
Landau and Lifshitz, *Classical theory of fields* and *Electrodynamics of continuous media*
Feynman lectures volumes 1–2 (free online: `www.feynmanlectures.caltech.edu/`.)
EM Purcell and DJ Morin, *Electricity and magnetism* (3rd ed.)
A Zangwill, *Modern Electrodynamics*

Tentative Outline

We can't cover all of this, so let me know if there's a topic you really want to see. See the weekly Assignments for reading and homework assignments.

Prologue

Gladstone: "What is the practical worth of electricity?"
Faraday: "One day, Sir, you may tax it."

0. **Parable: The equations of Newtonian gravity**

Hello Maxwell

Maxwell's theory is Maxwell's system of equations.... It is impossible to study this wonderful theory without feeling as if the mathematical equations had an independent life and an intelligence of their own, as if they were wiser than ourselves, indeed wiser than their discoverer, as if they gave forth more than he had put into them. – Heinrich Hertz

1. **What the equations say**
   Necessity of the field idea. Lorentz force law gives meaning to the quantities appearing in Maxwell.

2. **Static charges**
   The potential. Scattering in a potential; proton therapy. Field energy; capacitors. Linear, polarizable media; dielectric constant. Electrostatic multipole expansion. Force and torque on a rigid multipole in an external field. Earnshaw theorem. Separation of variables in Laplace's equation: Lightning rod and nearfield scanning optical microscopy probe. Fields outside a thin disk. Angular dependence of Förster resonance transfer (FRET).

3. **Electrostatics in water**
   Poisson–Boltzmann equation, Debye screening. Applications to macromolecules.

4. **Stationary but nonstatic sources**
   Charge density, current, and the continuity formula. Necessity of the "displacement current" term. Relaxation time scale, quasi-static case. Cell membrane capacitance measurement. Cable equation and its nonlinear modification; action potentials along neurons. Electrocardiogram. Extracellular recording from individual neurons.

5. **Dimensions and units.**
   SI sí, CGS no.

6. **Magnetostatics**
   Vector potential. Bohm–Aharonov effect: The vector potential is real. Current dipole field and magnetoencephalography. Magnetostatic multipole expansion. Force and torque on a dipole in external fields. Magnetic tweezers and magnetic cell sorting.

7. **Time dependence**
   Induced EMF means nonconservative electric fields. Self- and mutual inductance.
8. **Simplest wave solutions**
   Plane and spherical waves in vacuum. Linear and circular polarization. Reflection interference contrast microscopy. Waves transport momentum and energy.
9. **Gaussian beams and Bessel beams**
   Ordinary and optical-vortex beams.
10. **The ray-optics limit**
    Plane waves hitting a plane interface between two homogeneous media; Snell's rule. Optical tweezers. Total internal reflection: fiber-optics, endoscopy, TIRF microscopy. Generalization to arbitrary nonuniform medium; Eikonal approximation. Applications: mirages; reflection from the ionosphere; ophthalmic gonioscopy; radio telescopes.
11. **Diffraction**
    Stationary phase and the criterion for sharp shadows. Diffraction at a caustic. Application: Rayleigh limit in microscopes and telescopes.

Relativistic classical fields

"Oh, that Einstein, always cutting lectures—I really would not have believed him capable of it." – Einstein's former teacher Minkowski, upon reading Einstein's relativity paper.

12. **Lorentz invariance**
    Rotational invariance and tensors in 3D. Relativistic invariance and tensors in 4D. Doppler shift, aberration of starlight, Fizeau/Zeeman experiment. Dipole anisotropy of the cosmic microwave background radiation. An interlude on other tensor analyses: flavor, weak SU(2), and grand unified multiplets.
13. **Why Maxwell's equations look like that**
    Invariant form of Maxwell equations; need for the displacement term, and its sign. The essential role of charge conservation. 4-vector potential.
14. **Variational formulation**
    Lagrangian for EM fields, and for particles+fields. Noether theorem.
15. **Energy and momentum of fields**
    Energy–momentum tensor. Faraday's imagery and its mathematical implementation. Applications: stellar equilibrium, comet tails, Teller-Ulam mechanism, the early Universe. Beams with orbital photon angular momentum. Vista: Micromanipulation with light.
16. **Wave polarization, full and partial**
    Spin of the photon. Linear and circular polarization. Stokes parameters; Poincaré sphere.
17. **Waveguides and transmission lines**
    A ladder of lumped elements. Dispersion and cutoff. Boundary conditions at a perfect conductor. Modes in a rectangular or circular guide. Zero-mode guides and single-molecule biophysics.

Fiat Lux

"It is difficult to deal with an author whose mind is filled with a medium of so fickle and vibratory a nature...; We now dismiss...the feeble lucubrations of this author, in which we have searched without success for some traces of learning, acuteness, and ingenuity, that might compensate his evident deficiency in the powers of solid thinking..." – Henry Brougham, 1803 [Criticizing Thomas Young's wave theory of light]

18. **Radiation**
    An accelerated charge develops kinked field lines. Retarded potentials. Jefimenko formulas. Application to terahertz generators.

19. **Slowly-moving sources**
    Far fields of an oscillating electric dipole; magnetic dipole; higher multipoles. Defocused orientation and position imaging (DOPI). Antennas. How pulsars radiate. Near fields: near-field scanning optical microscopy.

20. **Rapidly-moving sources**
    Relativistic beaming. Liénard–Weichert potentials and fields. Cherenkov radiation. Bremsstrahlung and synchrotron radiation. Wigglers and undulators.

21. **Free electrons and waves**
    Single electrons: Thomson scattering and its polarization. Mean free path of radiation in the Sun. Scattering of unpolarized light; polarization of CMBR reveals quadrupole moment of the cosmic inhomogeneity.

22. **Waves in plasmas**
    Mean field approximation. Dispersion relation. The ionosphere: Why you can hear Europe. EM waves in metals; skin depth. Why pulsars seem to chirp. Faraday rotation in magnetized plasma; measurement of cosmic magnetic fields.

Light in matter

May: "Have the crystals faults like us?" Lily: "Certainly, May. Their best virtues are shown in fighting their faults. And some have a great many; and some are very naughty crystals indeed." – John Ruskin, 1866

23. **Bound electrons**
    Atomic polarizability and its classical representation. Permanent and induced moments. Anisotropy: the symmetric polarizability tensor. Single-dipole scattering, its frequency dependence, and its polarization. Return to the microwave polarizer and polaroid sheets. Blue, polarized sky versus white, unpolarized clouds.

24. **Effective form of Maxwell equations in dense matter**
    Polarization density. Bound charge and current densities. Clausius–Mossotti relation. Energy of fields+media. Energy–momentum flux tensor. Relativistic formulation.

25. **Waves in linear dielectric material**
    Dichroism and birefringence. The half-wave plate. Phase contrast microscopy. Evanescent wave; frustrated TIR. Total internal reflection fluorescence microscopy. Fresnel equations; Brewster angle. Birefringence and double refraction.

26. **Magnetic polarizability**
    The full polarizability tensor. Isotropic, but parity-noninvariant contributions; scattering from a helical wire. Optical activity in chiral media..

27. **Applications to biophysical chemistry**
    Circular dichroism and optical rotatory dispersion.